Problem 6.15 of Shuler & Kargi. Steady-state operation of a continuous bioreactor. Instructor: Nam Sun Wang

Conversion of substrate to product by an organism (with negligible biomass production)

$$S \longrightarrow P$$

Monod growth kinetics

$$\mu_{\rm m} = 1$$
 (h<sup>-1</sup>) K<sub>s</sub> = 0.01 (g/L)  $\mu(s) := \frac{\mu_{\rm m} s}{K_{\rm s} + s}$ 

Yield coefficient

 $Y_s = 0.5$  (g cell/g substrate)

Product formation

 $\alpha := 0.4$  (mg product/g cell)  $\beta := 0.5$  (h<sup>-1</sup>·mg product/g cell)

Operating condition

Dilution rate  $D = 0.8 \cdot \mu_m$   $D = 0.8 (h^{-1})$ 

Feed substrate concentration  $s_0 = 1$  (g/L)

Find steady-states (via analytical solutions)

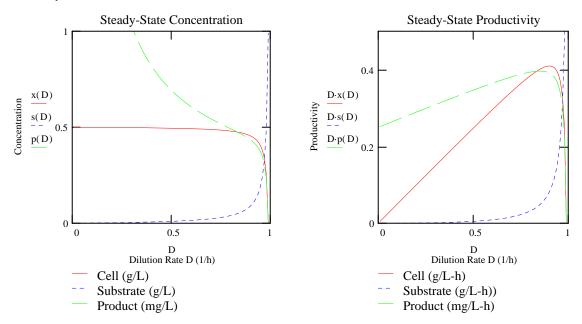
$$\frac{d}{dt} \mathbf{x} = \mathbf{0} = (\mu(s) - D) \cdot \mathbf{x} \longrightarrow \mu(s) = D \longrightarrow s(D) := K_s \cdot \frac{D}{\mu_m - D} \qquad s(D) = 0.04 \quad \text{(g)}$$

$$\frac{d}{dt} \mathbf{s} = \mathbf{0} = D \cdot \left(s_0 - s\right) - \frac{1}{Y_s} \cdot \mu(s) \cdot \mathbf{x} \longrightarrow x(D) := Y_s \cdot \left(s_0 - s(D)\right) \qquad x(D) = 0.48 \quad \text{(g)}$$

$$\frac{d}{dt} \mathbf{p} = \mathbf{0} = -D \cdot \mathbf{p} + \alpha \cdot \mu(s) \cdot \mathbf{x} + \beta \cdot \mathbf{x} \longrightarrow p(D) := \left(\alpha + \frac{\beta}{D}\right) \cdot \mathbf{x}(D) \qquad p(D) = 0.492 \quad \text{(mg/L)}$$

 $\label{eq:productivity} \mbox{ Productivity at the given } D \quad D \cdot p(D) = 0.394 \quad \mbox{ (mg product/h)}$ 

Plot steady-states D = 0.01, 0.02..0.99



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